Uspekhi Mat. Nauk 39:4 (1984), 85-94 Russian Math. Surveys 39:4 (1984), 99-110

Nikolai Nikolaevich Yanenko (obituary)

On 16 January 1984 the death occurred of Nikolai Nikolaevich Yanenko, Hero of Socialist Labour, holder of a State Prize of the USSR, Director of the Institute of Theoretical and Applied Mechanics of the Siberian Division of the Academy of Sciences of the USSR, an outstanding scientific administrator and broadly cultured man.

Yanenko was born in 1921 in the Novosibirsk region, into a peasant family. In 1939 he enrolled in the Faculty of Physics and Mathematics at the University of Tomsk and graduated in 1942, ahead of time and with distinction. From November 1942 onwards Yanenko took part in the Second World War. With a rifle in his hands he defended our country against the fascist aggressors, and for his display of courage and fortitude he was awarded the Order of the Red Star and medals.

In 1946 Yanenko became a postgraduate student at the Institute of Mathematics and Mechanics at the University of Moscow. Here he began his way into science under the supervision of the notable Soviet geometer and educator P.K. Rashevskii. In 1948 Yanenko presented his Ph.D. thesis and in 1954 his D.Sc. dissertation.

In 1948 Yanenko began work on applications of the methods of mathematical physics and computational mathematics to the solution of the most important scientific and technical problems of that time, under the supervision of A.N. Tikhonov. This was a turning point in his mathematical career. He, an "abstract geometer", began to work on the solution of applied problems. He had to study many branches of mechanics, physics, and computational mathematics, and the possibilities of computational techniques, while working at the same time on the solution of specific problems. Until 1955 Yanenko was also actively concerned with many-dimensional differential geometry. Despite the labour involved, Yanenko was brilliantly suited to these problems. For his great services in the accomplishment of a number of scientific and technical tasks Yanenko was awarded twice the Order of the Red Banner of Labour (1953, 1955) and was honoured with a State Prize of the USSR (1953).

During these years Yanenko became a highly qualified expert in computational mathematics, which at that time was taking its first steps, in mathematical physics, and in the theory of non-linear partial differential equations. His first published papers were devoted to computational methods and to the study of the properties of solutions of non-linear equations. He paid particular attention to the equations describing the behaviour of a solid medium under various conditions, and to numerical methods for their solution.

In 1956 Yanenko gathered and directed a large group of young pure and applied mathematicians and worked with them on the solution of systems of non-linear partial differential equations and on the development and perfection of computational methods for the solution of problems in mathematical physics.

From 1957 to 1963 he developed his now world-famous "method of fractional steps". This method makes it possible to reduce a computational solution of an initial many-dimensional problem to the consecutive solution of a number of one-dimensional problems. This turned out to be highly effective and reduces the length of time on an electronic computer by several orders of magnitude.

From 1963 until the end of his life Yanenko worked in the Siberian Division of the Academy of Sciences of the USSR, first at its Computational Centre, and from 1976 on as Director of its Institute of Theoretical and Applied Mechanics. Here in Siberia his talent and organizational abilities, his Siberian sense of purpose and his steadfastness, his Communist scholar's sense of principle, and his Soviet approach to solving major scientific problems showed themselves to the fullest extent. This period of his mathematical activity was also highly fruitful.

In Siberia Yanenko once again proceeded to select young scientists (pure and applied mathematicians, physicists) to solve important applied problems. A large scientific group was created by his efforts, which can rightfully be called the Siberian School of Academician Yanenko. The range of his scientific interests broadened radically. He became occupied with mathematical modelling of real processes, both from the theoretical and the purely practical point of view. He was interested in questions of the organization of modelling, programming, applications of electronic computers and the development of a stock of machinery; and in the construction of packages of applied programmes. He actively studied a number of problems in mechanics and technology.

Yanenko also continued intensive work along lines that were traditional for him: systems of non-linear equations, and the theory of shock waves; the theory of difference methods; and models in the mechanics of continuous media. In 1967 his monograph "The method of fractional steps in the solution of many-dimensional problems in mathematical physics", appeared and was soon translated into German, French, and English. His monograph "Systems of quasilinear equations and their applications to gas dynamics" (1968), written jointly with B.L. Rozhdestvenskii, received wide recognition. It contains the results of many years of work by the authors and led up to the sum total of research by scholars the whole world over in the theory of systems of non-linear equations of hyperbolic type with two independent variables. The second edition (1978) reflected progress in this area over the previous decade.

Such concepts as the "approximative viscosity of a difference scheme" and the "first and higher-rank differential approximations to a difference scheme" were developed mainly by Yanenko and his students. In 1973 he began to study essentially non-linear equations that change their type depending on the solution. The large range of papers on this topic by Yanenko and his pupils is reflected in the monograph "Non-linear equations of variable type" (1983). We should mention the work of Yanenko and his pupils on refining the results of numerical calculations of problems in gas dynamics by the method of the "differential analyser"; on the regularization of the Navier-Stokes equations for an incompressible fluid; and on the optimal choice of difference nets.

Yanenko is the author of more than 300 scientific works, including eight monographs. Clearly, even a brief survey of his work requires a thorough study. Therefore, we can consider below only some areas of his work that characterizes him as a mathematician.

Yanenko attached great importance to the administration of research in important areas of science. He conducted six periodic All-Union scientific Seminars.

We name two especially popular seminars of his: "Computational methods on the mechanics of a viscous fluid" and "Models in the mechanics of continuous media". He was the initiator in the founding of and the managing editor of the periodic collection "Computational methods in the mechanics of a continuous medium", and was a member of the editorial board of several Soviet scientific journals.

Yanenko had great international scientific prestige. A member of the working group IFIP and of the IUTAM office, and a member of the editorial board of three international journals, he was when abroad always a travelling propagandist for Soviet science. He gave brilliant lectures at the Sorbonne and at Cambridge. In this he was aided by his excellent knowledge of five foreign languages. He gave much attention to strengthening international scientific links, especially with our fellow socialist countries.

Yanenko considered his unceasing teaching work to be a necessary part of his scientific work. He worked at the Universities of Moscow, the Urals, and Novosibirsk; he became a professor (in 1960), holding the chair of computational methods in the mechanics of a continuous medium at the University of Novosibirsk. He gave special attention to training scientific staff for higher qualifications; among his direct students, there are eight D.Sc.'s, and fifty Ph.D.'s.

His enormous scientific, public, and international work did not obscure his exceptional character as a person. He was natural, intelligent in the true sense, and deeply principled. Consideration and simplicity, kindness, and constant readiness to help were characteristic features of his relationships with friends, colleagues, and students. Together with this, a genuine courage and fighting steadfastness were concealed under an outward gentleness and disinclination to be in the centre of attention. Approachable by all—from the fellow academician to the student—he devoted all his numerous talents to the maximum service to science.

Yanenko's fruitful many-sided activity were also highly valued during the period of his work in Siberia. In 1966 he was elected a Corresponding Member and in 1970 an Active Member of the Academy of Sciences of the USSR; he was awarded Orders of the Red Banner of Labour and of the October Revolution; and he was repeatedly honoured as a Laureate of the State Prize of the USSR. On 22 May 1981 Yanenko was awarded the high title of Hero of Socialist Labour.

During the last years of his life Yanenko worked especially strenuously, taking no account of time and surmounting ill-health. Death cut short his life in the bloom of his creative powers, at a moment when a great many important tasks had been accomplished, but when an even larger number of tasks had only begun ...

The bright memory of Nikolai Nikolaevich Yanenko is cherished in the hearts of his students, friends, and comrades. He will always remain an ideal of modern man: a scholar, citizen, and patriot.

* * *

Yanenko's geometrical research is reflected in thirteen publications and his Ph.D and D.Sc. dissertations.(1) His first results concern a classical problem of differential geometry: the problem of *deformation* of surfaces (in its local aspect). It had already been shown in papers by the geometers of the last century that a surface in three-dimensional Euclidean space admits a continuous deformation. Attempts to prove such a result for a hyperplane in E_n, $n > 3$, were unsuccessful; moreover, it was discovered (Bitz 1876) that the hyperplane $S_n \subset E_{n+1}$ of rank $r > 2$ is non-deformable. (The rank of a hyperplane is the dimension of the manifold of its tangent hyperplanes.) More recently Sbran (1909) and Cartan (1917) gave a complete classification of deformable hypersurfaces. The natural next step was the study of deformable surfaces $S_n \subset E_{n+q}$ for $q > 1$. The problem turned out to be exceptionally difficult, and the first substantial result did not follow until 1939: Allendörfer, by introducing the concept of the type of a surface (an arithmetical invariant of an embedding defined in a complicated analytic manner) proved that a surface $S_n \subset E_{n+q}$ is non-deformable if its type t is

(1)Ph.D. dissertation: On some necessary criteria for deformable surfaces in an n-dimensional Euclidean space, Moscow, Inst. Math. Moscow University, 1948. D.Sc. dissertation: On the theory of embedding Riemann metrics in a many-dimensional Euclidean space, Moscow University (Library) 1954.

greater than 2. Thus, the condition $t \leqslant 2$ is necessary (although far from sufficient) for a deformation. Yanenko succeeded in finding a number of deep necessary criteria for a deformation of S_n in E_{n+q}. By studying the structure of the deformation equations (the Gauss, Peterson-Codazzi and Ricci equations), he was able to obtain a number of effective results. Here is one of them: if there are two surfaces S_n' and S_n'' that are isometric to S_n but not congruent to each other, then there exists a continuous deformation from S_n' to S_n'' preserving the metric.

The problem of deformation of surfaces is closely connected with that of *embedding* a Riemann manifold in a Euclidean space (here we again have in mind the local aspect). For, the deformation problem concerns the *number of solutions* of the embedding equations and the connections between the solutions, while the embedding problem concerns the *existence* of a solution. In the embedding problem, Yanenko's results were perhaps the most impressive after the work of Cartan, Thomas, and Allendörfer. Considering the embedding equations of S_n in E_{n+q} Yanenko established necessary conditions under which all these equations are algebraic consequences of only one of them: the Gauss equation

$$R_{ij,kl} = \sum_{s=1}^{q} (\lambda_{ik}^{(s)}\lambda_{jl}^{(s)} - \lambda_{il}^{(s)}\lambda_{jk}^{(s)}),$$

where $\lambda_{pq}^{(s)} = \lambda_{qp}^{(s)}$ are the unknowns. From this it follows that when these necessary conditions are satisfied, the solution of the problem of embedding S_n in E_{n+q} reduces to the purely algebraic task of determining the ranks of certain matrices.

We cannot do justice in a brief form to the scale of Yanenko's work on the determination of the class of a Riemannian space V_n (the minimum value of q for which an embedding of V_n in E_{n+q} is possible). Therefore, we restrict ourselves to describing only two results (chosen for reasons of the simplicity of stating them). We consider the series of integral invariants $T_1, T_2 \ldots$ of the curvature tensor of the metric of V_n (here T_1 is the rank of the curvature tensor); it can be shown that 1) if $T_q > 2$, then the class of the metric cannot be less than q; 2) if $T_q \geqslant 4$, then a necessary and sufficient condition for the class to be q is the solubility of the Gauss equations.

There is a fuller account of Yanenko's geometric results in his survey articles.(1)

Non-linear partial differential equations and their solutions occupied Yanenko's interest from the early 50's until the end of his life. In the middle of our century the rapid development of science and technology, including computer technology, necessitated a systematic application of non-

(1)Trudy Sem. Vektor. Tenzor. Anal. **1952**, no. 9, 236-287; **1956**, no. 10, 139-191; Uspekhi Mat. Nauk 8:1 (1953), 21-100; Trudy Moskov. Mat. Obshch. **3** (1954), 89-180.

linear equations to the description of numerous real physical phenomena; the calculation of their solutions; and the application of the results to the construction and development of new forms of technology. At once one discovered the insufficiency of our knowledge about non-linear equations and their solutions, and the lack of rigorous and approximate theories that would make it possible to calculate non-linear processes. In particular, especially acute became the problem of creating a mathematical theory of shock waves, and in a broader sense, a mathematical theory of discontinuous "solutions" of systems of non-linear partial differential equations. The most outstanding mathematicians were involved in work on the creation of a mathematical apparatus corresponding to these needs of science.

Yanenko also made a substantial contribution to the development of the theory of non-linear equations of mathematical physics and computational methods for solving them. He published about fifty works on the theory of non-linear equations, including three monographs, and an even greater number of papers on computational methods for solving them.

His first papers on the theory of systems of quasilinear equations of hyperbolic type were published in 1955 in the Uspekhi Matematicheskikh Nauk. In one of them he considered the problem of what essentially non-linear systems of two quasilinear equations of hyperbolic type have the property of preserving smoothness of their solutions, in particular, in the solutions of the Cauchy problem for which discontinuities are not formed if the initial values do not have them. He found a special class of such systems; later systems of quasilinear equations of this type became known as "weakly non-linear".

He introduced methods developed in geometry to the study of systems of quasilinear equations. He developed the so-called "method of differential connections" for selecting classes and searching for individual solutions of systems of differential equations.

He stated the basic idea of this method at the Fourth All-Union Mathematical Congress. It consists in the selection of particular solutions of a system of differential equations

$$(S) \qquad S(x, u, p) = 0$$

(where x and u are vectors; x are the independent and u the dependent variables; the p are products of u's and x's) proceeding by adjoining to it a system of supplementary differential relations

$$(D) \qquad D(x, u, p) = 0.$$

The resulting overdetermined system (SD) is investigated for compatibility. If (SD) is compatible, then one can look for its solutions, which are also solutions of (S). The arbitrariness of the general solution of (SD) is less than that of (S), therefore, it is simpler to find them. The arbitrariness of the solution of the given system (S) is successively narrowed down by adding an even greater number of differential connections (D) to it. Here connections of the most general form are taken and it is required that the resulting overdetermined systems (*SD*) are in involution.

The standard automodel solutions and solutions with a degenerated hodograph can be obtained by this method. By means of the method of differential connections Yanenko and his students solved a number of interesting problems in one-dimensional and many-dimensional gas dynamics and the motion of an inelastic continuous medium.

These and a number of other problems linked with the method of differential connections are set forth in detail in Yanenko's monograph,[1] which was not published until after his death.

A stage in Yanenko's work on non-linear equations was the publication in 1968 of a monograph,[2] which was the fruit of many years of work on the systematization and an exposition from a general point of view of the numerous results obtained all over the world up to that time in the theory of systems of quasilinear equations of hyperbolic type with two independent variables. It reflected progress in the study both of *classical* and of *generalized* (discontinuous) solutions of such systems, in the theory of shock waves and the solution of many important problems of gas dynamics, as well as in the development of difference methods of solving problems of gas dynamics. This monograph is well known to experts and is widely used by pure and applied mathematicians and physicists. It is unique in the world literature as a book that gives a fairly full account of the present state of research in this important field. The second edition of this monograph in 1978 is substantially rewritten and reflects the progress during the preceding decade; it was translated into English by the American Mathematical Society.

Yanenko personally made a substantial contribution to the contents of the monograph: he presented a number of new accounts of known results. We mention, in particular, that the systematization of the construction of a solution to the problem "on the dissolution of an arbitrary discontinuity", one of the most important problems in gas dynamics, is due to Yanenko. He treats the motion arising as a result of the "dissolution of a discontinuity", as the result of the motion of a piston simultaneously in two contiguous media. Here he was the first to give a rigorous mathematical proof of the existence and uniqueness of the solution of the problem "on the dissolution of an arbitrary discontinuity" for a broad class of thermodynamic characteristics of contiguous gases.

From 1964 onwards in the Central Research Institute Siberian Branch of the Academy of Sciences of the USSR investigations were carried out under

(1) A.F. Sidorov, V.P. Shaplev, and N.N. Yanenko, The method of differential connections and its applications to gas dynamics, Nauka, Novosibirsk 1984.

(2) B.L. Rozhdestvenskii and N.N. Yanenko, Systems of quasilinear equations and their applications to gas dynamics, Nauka, Moscow 1968; second ed. 1978.

Yanenko's direction on the computational solution of problems in the dynamics of a viscous incompressible fluid. As is well-known, the Navier-Stokes equations in the case of an incompressible fluid are not of Cauchy-Kovalevski type.

In this connection, approximately in 1965 Yanenko had the idea of replacing the Navier-Stokes equations for an incompressible fluid by equations of Cauchy-Kovalevski type with a small parameter ε such that as ε tends to zero, the approximating equations turn into the original ones. In the mathematical scheme approximating the initial and boundary-value problem for the Navier-Stokes equations reduces, for example, to the solution of the system of equations

$$\frac{\partial v}{\partial t}+(v\cdot\operatorname{grad})\,v+\frac{1}{2}\,v\operatorname{div}v+\operatorname{grad}p=\operatorname{div}\operatorname{grad}v+f,$$

$$\varepsilon\frac{\partial p}{\partial t}+\varepsilon_1 p+\operatorname{div}v=0,\quad t>0,\quad x\in Q,$$

under the conditions

$$v=v_0(x),\ p=p_0(x),\ \operatorname{div}v_0=0,\ t=0,\ x\in Q;$$
$$v=0,\ t>0,\ x\in\partial Q.$$

Yanenko and his students, and later a number of mathematicians at home and abroad, proved that as ε and ε_1 tend to zero, the solution of the ε-problem tends in a certain sense to the solution of the original problem of the dynamics of a viscous fluid.

In 1973 Yanenko considered the equation

$$(*)\qquad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}\left[\nu\left(\frac{\partial u}{\partial x}\right)\cdot\frac{\partial u}{\partial x}\right]$$

with a coefficient $\nu(p)$ that changes sign and is asymptotically positive as $|p|\to\infty$. This equation is a generalization of the standard Buergers equation, which plays an important role in the study of properties of solutions of the equations of gas dynamics.

The equation $(*)$ was introduced with the aim of modelling such complex phenomena as the loss of stability, the presence of auto-oscillations, or intermittence arising in the motion of a viscous liquid. It is characterized by the fact that in its solution a change can occur in the direction of parabolicity, and it is in essence a new mathematical object. Various boundary-value problems for the equation $(*)$ were studied both by analytical and by numerical methods. Theorems were proved on the existence and non-existence of solutions in various situations, results were obtained on the uniqueness and non-uniqueness, and problems of stabilization as $t\to\infty$ were studied. Auto-oscillation solutions for the equation $(*)$ were constructed, as well as solutions of shock transfer type. By means of direct numerical modelling, the stability both of stationary and of auto-oscillating solutions were studied. Apart from the equation $(*)$, its two-dimensional analogue was also studied, that is, an equation of Navier-Stokes type with a sign-changing coefficient of viscosity. Such equations occur in the search for the mean velocity profile of a turbulent flow.

In a number of papers Yanenko studied equations of the form

$$(**) \qquad \frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left[F\left(\frac{\partial u}{\partial x}\right)\right]+\frac{\partial^2}{\partial x\,\partial t}\left[F_1\left(\frac{\partial u}{\partial x}\right)\right],$$

where it is assumed concerning F that F' may change sign. These equations are regularizations of $(*)$ and are of interest in their own right, in as much as they arise in modelling the processes that occur during the growth of materials, in media with memory, and in gas dynamics. Under the condition that F is subordinate to F_1, existence and uniqueness theorems were proved for the solutions of the basic boundary-value problems for the equation $(**)$.

The results of Yanenko and his students in the study of equations of variable type were summarized in the last monograph(1) of his life.

Difference methods and computational mathematics occupied Yanenko from 1949 onwards. His first paper on this topic was completed in 1951, and all in all there are more than 150 papers, including his widely known monograph(2), lectures for collaborators and students, and papers on both general and specific problems of computational mathematics.

One of Yanenko's main achievements in computational mathematics is the creation of the "method of fractional steps". The idea of creating simple and economic methods for solving many-dimensional problems by reducing them to a one-dimensional routine, was developed in a number of papers by Soviet (Saul'ev) and American (Peaceman, Rechford, Douglas) mathematicians. Yanenko's contribution consists in the fact that in contrast to previous work he was the first (1959) to formulate a method of reducing a many-dimensional problem to a collection of one-dimensional problems.

For example, the difference equation approximating the equation of heat conduction in k-dimensional space,

$$(*) \qquad \frac{u^{n+1}-u^n}{\tau}=\sum_{s=1}^{k}\Lambda_s u^{n+1}$$

(where the Λ_s are one-dimensional difference operators approximating the operators $\frac{\partial^2}{\partial x_s^2}$) requires for the determination of u^{n+1} a solution of a system of linear equations with a large number of unknowns. Yanenko replaces the difference equation $(*)$ by a collection of k difference equations

$$(**) \qquad \frac{u^{n+s/k}-u^{n+(s-1)/k}}{\tau}=\Lambda_s u^{n+s/k}, \quad s=1,2,\ldots,k.$$

(1)N.A. Larkin, V.A. Novikov, and N.N. Yanenko, Non-linear equations of variable type, Nauka, Novosibirsk 1983.

(2)N.N. Yanenko, The method of fractional steps for solving many-dimensional problems of mathematical physics, Nauka, Novosibirsk 1967.

Each of the (**) splits into a collection of one-dimensional equations, which are easy to solve by "turning a handle". Such a "splitting" is a reflection of the additivity of physical processes and of the spatial operators describing them.

The representation (**) was a bold and innovative assumption, which made it possible to approach in a new way many problems in the theory of difference schemes. It stimulated further progress in the development of the theory of difference schemes for many-dimensional and even for one-dimensional problems. He introduced the generalized concepts of a summary approximation and of a weak approximation of a many-dimensional equation by a system of one-dimensional ones. This made it possible to carry out "a splitting" of difference equations not only with respect to the independent variables, but also the various physical processes and the individual terms of differential and difference equations with the aim of simplifying the solution of difference equations. We mention the deep connection between methods of splitting with the problems of the stability of the solution of difference equations and methods of solving them.

In 1968 Yanenko turned to the solution of many-dimensional Navier-Stokes equations for a compressible gas, regarding them as a more complete model for the study of complex processes in hydro- and aerodynamics. The analysis of these solutions led him to the necessity of extending the concept of splitting and its formulation as a method of solving complex many-dimensional problems in mathematical physics. His form of a geometric, a physical and an analytic splitting served as a basis for the construction of difference schemes for a broad class of problems in mathematical physics.

The application of the Navier-Stokes equations to the solution of problems in complicated domains required the development of new methods of constructing design nets and working out the variation principle of net control. Supplementing the initial equation with equations for net control, Yanenko regarded the construction of a net as a problem of constructing a differential map, corresponding to a globally constant flow. This led him to the concept of an information medium as a set of differential equations describing physical processes and equations for controlling a net, automatically adapting to a flow.

He considered the problem of developing effective numerical methods for solving non-linear equations of the mechanics of a continuous medium, together with the problem of an effective use of numerical results obtained on an electronic computer. In particular, for the solution of problems of gas dynamics with the use of finite-difference schemes of "through count" he proposed to determine the position of shock waves in "spread-out" profiles of a numerical solution with the aid of special algorithms, which he called differential analyzers. At the beginning of the 60's, on the basis of these algorithms, he proposed to define the fronts of a wave as the points at which the magnitude of artificial viscosity attains its maximum. He was the initiator of research on the problem of working out the algorithms of differential analyzers and a number of other algorithms of localization of singularities in the numerical solution of problems of gas dynamics. By means of his methods he studied the standard ways of localizing discontinuities in numerical solutions of problems of gas dynamics, and he established conditions for their applicability. The algorithms of differential analyzers were applied in the numerical study of a number of applied problems.

As far back as the 60's Yanenko described a method of studying the properties of algorithms for the numerical solution of problems in mathematical physics, the so-called method of differential approximations to a difference scheme. This method was widely developed in papers by Yanenko and his students, and continues to be developed even now both in the USSR and abroad. The basic idea of this method consists in replacing the study of properties of a difference scheme by that of a certain problem with differential equations occupying an intermediate position between the original differential problem and the difference scheme approximating it.

For example, let us consider the explicit difference scheme

$$(*) \qquad \frac{u^{m+1}(x)-u^m(x)}{\tau} = \Lambda_0(h(\tau))\, u^m(x),$$

where $u^m(x) = \bar{u}(m\tau, x)$ is a difference analogue to the solution of the differential vector problem $v(t, x)$, τ is the step in time, and $\Lambda_0(h(\tau))$ is a certain difference operator that depends analytically on $\tau > 0$. The solution of the scheme $(*)$ can be written in the form

$$(**) \qquad u^m(x) = \bar{u}(t, x) - (E + \tau\Lambda_0)^{t/\tau}\, u^0(x),$$

where $t = m\tau$. We take the analytic continuation of the representation $\bar{u}(t, x)$ in the form $(**)$ for all t, $\tau > 0$. By differentiating $(**)$ with respect to t we obtain the differential equation

$$\frac{\partial \bar{u}}{\partial t} = \frac{1}{\tau}\log[E + \tau\Lambda_0]\,\bar{u}$$

and decomposing it with respect to τ with $\|\tau\Lambda_0\| < 1$, we arrive at the following representation of the difference scheme:

$$(***) \qquad \frac{\partial \bar{u}(t, x)}{\partial t} = \Big[\Lambda_0 + \sum_{n=1}^{\infty} (-1)^n \cdot \frac{\tau^n}{n+1}\Lambda_0^n\Big]\,\bar{u}.$$

By discarding higher terms on the right-hand side of the series, we obtain differential approximations to the difference scheme $(*)$ of various orders. Here one has to take into account the dependence of A_0 on τ. Finally, if we represent $A_0(h(\tau))$ in the form of a series containing the differentiation operator with respect to the spatial coordinates x (which is usually possible for difference schemes), and discarding in it only the higher terms in τ, then we obtain differential approximations of the scheme $(*)$ in the form of partial differential equations. By studying these equations we can draw definite conclusions about the properties of the difference scheme (*) such as stability, approximation, exactness, the group property, etc.

Yanenko considered this method as a tool not only for the study, but also for the construction of difference schemes. The literature on the method of differential approximation has at present more than two hundred titles. The majority of the results in these papers have a heuristic character, but they are corroborated by numerous calculations. Rigorous results have been obtained by Yanenko's students.

* * *

In concluding this brief survey of Yanenko's scientific work, let us emphasize once again that many important areas of his research have been left out of our account. Nikolai Nikolaevich Yanenko left a rich scientific legacy, which needs serious study. There is no doubt that his numerous students will continue and develop all his very important work and thought.

V.G. Dulov, S.P. Novikov,
L.V. Ovsyannikov, B.L. Rozhdestvenskii,
A.A. Samarskii, Yu.I. Shokin

Translated by A. Dowker